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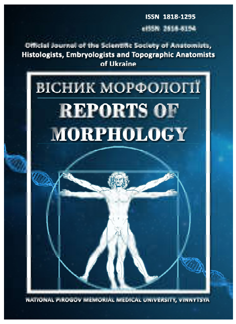

Fractal analysis of anatomical structures linear contours: modified Caliper method vs Box counting method

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Fractal analysis estimates the metric dimension and complexity of the spatial configuration of different anatomical structures. This allows the use of this mathematical method for morphometry in morphology and clinical medicine. Two methods of fractal analysis are most often used for fractal analysis of linear fractal objects: the Box counting method (Grid method) and the Caliper method (Richardson's method, Perimeter stepping method, Ruler method, Divider dimension, Compass dimension, Yard stick method). The aim of the research is a comparative analysis of two methods of fractal analysis - Box counting method and author's modification of Caliper method for fractal analysis of linear contours of anatomical structures. A fractal analysis of three linear fractals was performed: an artificial fractal - a Koch snowflake and two natural fractals - the outer contours of the pial surface of the human cerebellar vermis cortex and the cortex of the cerebral hemispheres. Fractal analysis was performed using the Box counting method and the author's modification of the Caliper method. The values of the fractal dimension of the artificial linear fractal (Koch snowflakes) obtained by the Caliper method coincide with the true value of the fractal dimension of this fractal, but the values of the fractal dimension obtained by the Box counting method do not match the true value of the fractal dimension. Therefore, fractal analysis of linear fractals using the Caliper method allows you to get more accurate results than the Box counting method. The values of the fractal dimension of artificial and natural fractals, calculated using the Box counting method, decrease with increasing image size and resolution; when using the Caliper method, fractal dimension values do not depend on these image parameters. The values of the fractal dimension of linear fractals, calculated using the Box counting method, increase with increasing width of the linear contour; the values calculated using the Caliper method do not depend on the contour line width. Thus, for the fractal analysis of linear fractals, preference should be given to the Caliper method and its modifications.

Keywords: *fractal analysis, morphometry, linear contour, Caliper, Box counting.*

Introduction

Fractal analysis has been increasingly used as a morphometric method in morphology and clinical medicine for the last few decades [3, 4]. This method of mathematical analysis provides a quantitative assessment of the metric dimension and complexity of the spatial configuration of different anatomical structures [12, 13]. Fractal analysis is based on fractal geometry, which characterizes the structure and spatial organization of fractals [12, 13, 14]. A fractal is a mathematical set or object characterized by self-repetition, self-similarity and large-scale invariance (part of the object partially or completely repeats the structure of the object as a whole, the structure of the object at different scales is similar) [1, 10, 12, 13, 14].

Fractals based on clear mathematical algorithms are called mathematical or artificial. Some natural objects (including the anatomical structures of the human body) have the properties of fractals, but their structure, unlike artificial fractals, is not mathematically regulated. Such structures are called natural fractals or quasi-fractals [12, 13, 14]. Artificial and natural fractals can be different in structure. Among the structures of the human body are often linear fractals, most often linear contours of various objects and structures, which in two-dimensional images are represented by curved or broken lines of different configurations (fibers, membrane cross-sections, outer and inner linear contours of various structures and pathological cells, etc.). Fractal analysis of linear contours of anatomical structures (outer contour characterizing the surface of the anatomical structure or inner contour characterizing the inner surface of a hollow organ or cavity inside the anatomical structure) allows to quantify the features of their spatial configuration: the more complex the linear contour of the formation (for example, the contour has a wavy, twisted, broken configuration, etc.), the more complex is the spatial configuration of the anatomical structure as a whole.

Fractal properties of different objects can be quantified using fractal dimension (FD, fractal index). The fractal dimension determined on two-dimensional images can vary from 1 to 2 [5, 6, 12, 13, 14]. Box counting (Grid Method) is most often used for fractal analysis of linear objects in medicine and morphology due to its simplicity and versatility [2, 7, 8, 18, 21]. In addition, the classic method used for fractal analysis of linear contours is the Caliper method (Richardson's method, Perimeter stepping method, Ruler method, Divider dimension, Compass dimension, Yard stick method) [5, 6, 11, 17, 19, 20, 23, 24]. However, in medicine, due to routine and lack of accuracy, this method is used much less often than the method of counting squares [11, 23]. In some studies, both methods of fractal analysis (Box counting and Caliper) were used in different modifications [17, 19, 20, 24] and a comparative analysis of Box counting and other methods of fractal analysis was performed [5, 6].

We developed our own modification of the Caliper method, adapted for use in morphology as a morphometric method [15] and used it for fractal analysis of the linear contour of the cerebellum [16]. In this paper, a comparison of two methods of fractal analysis for the selection of optimal techniques for morphometric study of linear contours of anatomical structures.

The aim of the study is a comparative analysis of two methods of fractal analysis - Box counting method and the author's modification of the Caliper method for fractal analysis of linear contours of anatomical structures.

Materials and methods

The study was conducted in compliance with the basic bioethical provisions of the Council of Europe Convention on Human Rights and Biomedicine (April 4, 1997), the Helsinki Declaration of the World Medical Association on Ethical Principles for Human Scientific Research (1964-2008), as well as the order of the Ministry of Health of Ukraine №690 of 23.09.2009. **The conclusion of the Commission on Ethics and Bioethics** of Kharkiv National Medical University confirms that the study was conducted in compliance with human rights, in accordance with current legislation in Ukraine, meets international ethical requirements and does not violate ethical standards in science and standards for conducting biomedical research (minutes of the meeting of the Commission on Ethics and Bioethics of KhNMU №10 dated November 7, 2018).

Three linear fractal objects were chosen as objects for comparative analysis of two methods of fractal analysis (Fig. 1): artificial (mathematical) fractal - Koch snowflake, which is a classic example of a linear fractal [1, 10, 12, 13, 14] (the fourth iteration was chosen for the study), and two natural fractal objects - the outer contours of the pial surface of the human cerebellar cortex and the cortex of the cerebral hemispheres. The pial surface of the cerebral cortex and the surface of the brain are traditionally considered as a self-similar complex fractal structure [9], and its study is of great importance for clinical neuroscience [3, 4, 7, 8], so different areas of the cerebral cortex (namely - its external linear contours on two-dimensional MR images) were selected for the study.

For fractal analysis of the external linear contours of the cerebellar cortex and cerebral hemispheres, magnetic resonance (MR) tomograms of the brains of persons who did not have structural changes in the brain were used.

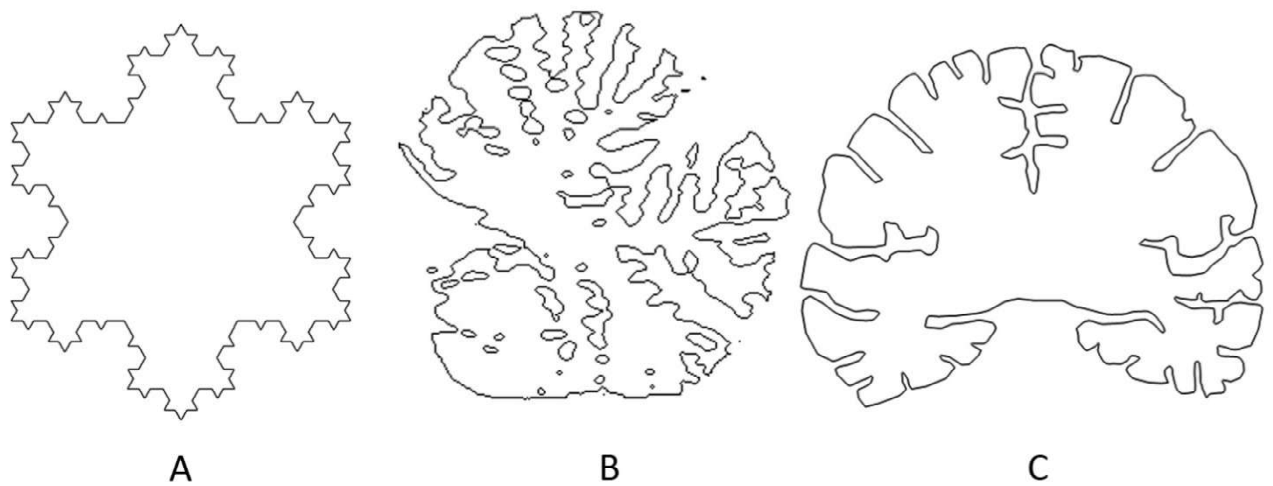


Fig. 1. Linear fractal objects used for fractal analysis: A - Koch snowflake, B - outer linear contour of the human cerebellar cortex, C - outer linear contour of the cortex of the human cerebral hemispheres.

Magnetic resonance imaging was performed using a magnetic resonance imaging with a magnetic induction value of 1.5 T. MR images of the cerebellum in sagittal projection were used for fractal analysis of the linear contour of the cerebellar vermis, and MR images in coronal projection were used to analyze the contour of the cerebral cortex.

Preliminary preparation of digital images for the two methods of fractal analysis (Caliper and Box counting) was performed in the same way. Adobe Photoshop CS5 created images with a resolution of 128 pixels per inch with the following dimensions: to study the contour of the cerebellum and Koch's snowflake, the image size was 128x128 pixels, the contour of large hemispheres - 512x400 pixels. A fragment of a digital MR image of the brain containing the structure under study, or a digital image of a Koch snowflake, was placed in the resulting rectangle. Subsequently, to determine the effect of scale and resolution on the fractal dimension, the image resolution was doubled (from 128 to 256 pixels per inch) and quadrupled (up to 512 pixels per inch), with linear image sizes varying in proportion to the resolution.

After preliminary preparation, fractal image analysis was performed. The methods of the two fractal analysis methods analyzed in this paper were described earlier: the Box counting method is a classic method used in the vast majority of studies using fractal analysis [2, 5, 6, 7, 8, 18, 21], the author's modification of the Caliper method described in our previous work [15] and used to study the cerebellum [16]. But we consider it expedient to give a general description of the two methods of fractal analysis in this paper to better understand the results.

Different methods of fractal analysis involve the use of fractal measurement units (fractal measurement units), which cover the object under study [5, 6, 12, 13, 14, 15, 18, 20, 21, 24]. Fractal measures can be linear segments, squares, cubes and other geometric shapes. The type of fractal measure is determined by the peculiarities of the method and the object under study. Fractal analysis always includes several stages, in which an iterative (repeated and uniform) change in the size of the fractal measure (S) is performed; most often it is a doubling or halving of the linear size of a fractal measure (for example, doubling or halving the length of a fractal linear segment or each side of a fractal square). At each stage of fractal analysis, the value of N is calculated - the minimum number of fractal measures that allow you to completely cover (cover, fit) the object under study. Then the natural logarithms of two numbers are calculated: N and 1/S - numbers, the inverse of the fractal measure (Ln (N) and Ln (1/S)). Then calculate the linear regression equation of the dependence of Ln(N) on Ln (1/S), the value of the fractal dimension is equal to the slope of the direct regression relative to the abscissa [5, 6, 12, 13, 14, 15, 16]. The value of the fractal dimension can be calculated by the formula:

$$FD = \frac{\Sigma\left(Ln\left(\frac{1}{S}\right) - \overline{Ln\left(\frac{1}{S}\right)}\right)\left(Ln(N) - \overline{Ln(N)}\right)}{\Sigma\left(Ln\left(\frac{1}{S}\right) - \overline{Ln\left(\frac{1}{S}\right)}\right)^2}$$

where FD is the fractal dimension, S is the relative size of the fractal measure, N is the number of fractal measures covering the structure under study (cited according to [16] with changes).

But, despite the similarities, the two methods of fractal analysis have fundamental differences.

Method of counting squares (Box counting). For fractal analysis with the help of the Box counting method an additional stage of preliminary image preparation is performed: with the help of a graphic editor a linear contour is selected and a line is drawn, which is necessary for further image analysis. Since the contour is studied, and not the structure as a whole, it is advisable to use the outline of the contour with a line of the minimum possible width, which for digital raster images is 1 pixel (Fig. 2). In addition, a 2-6 pixel-wide contour outline was used to determine the effect of linear contour width on fractal dimension values.

After delineation, the fractal analysis is performed according to the classical method of Box counting [5, 6]. A fractal grid is superimposed on the image, which divides the image into rectangles. In the first stage, the fractal grid lines divide each side of the image in half, and the size of the fractal measure (S) at this stage is 1/2. The size of the sides of the squares of the fractal grid decreases several times, so the value of S in the second stage is 1/4, the third - 1/8, the fourth - 1/16, the fifth - 1/32 (see Fig. 2). The fractal measure size for the Box counting method is also called box size (box edge size) [5, 6].

At each stage of fractal analysis, the number of fractal measures covering the contour (N) is determined by counting the number of squares containing fragments of the studied structure (in the study of delineated images - the contour of the studied structure) (see Fig. 2) [5, 6].

Fractal analysis using the method of Box counting in this study was performed automatically using the program Image J [22], the following values of S (box size) were selected: 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128.

Caliper method. The classic version of the Caliper method uses a one-dimensional fractal measure - a linear segment. A linear object is covered with a broken line consisting of linear segments of a certain length and the number of these segments is counted (N). Then their length is increased or decreased twice and the number of fractal measures is counted again [5, 6, 12, 13, 14, 15, 20, 21, 24]. This method in the classic version is routine, because the calculation is done manually.

In our proposed author's modification of the Caliper method [15, 16], the analysis is performed automatically using Adobe Photoshop CS5. This technique includes the following steps. After preliminary preparation on the investigated image the linear contour by means of the tool "selection" is allocated. Unlike the method of counting squares, this method does not require outlining a line. After selection, the length of the contour in pixels (P) is measured using the tool "analysis" (Fig. 3) [15, 16].

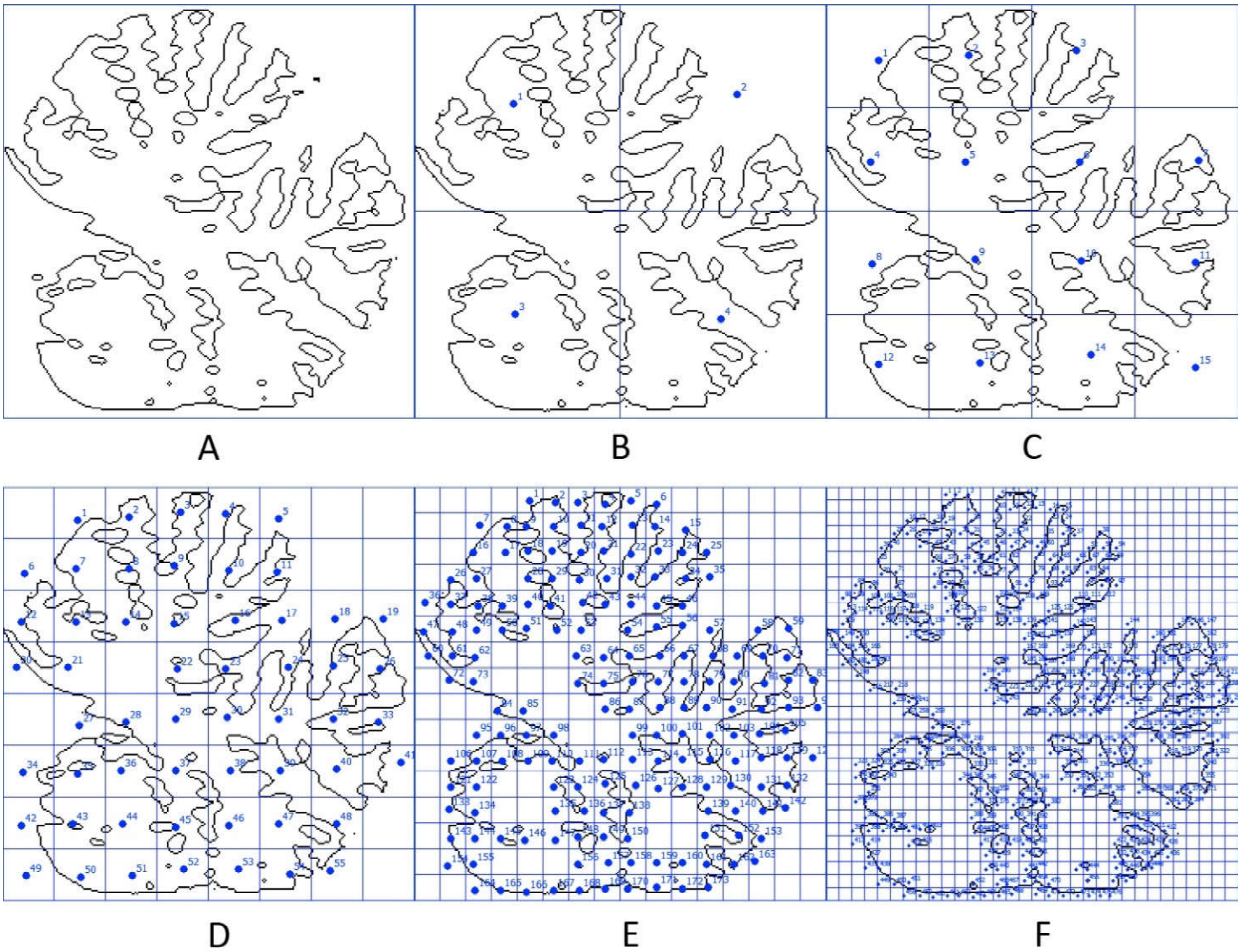


Fig. 2. Fractal analysis of the linear contour of the cerebellum using the method of Box counting. A - delineation of the linear contour of the object under study; B-F - stages of fractal analysis using a fractal grid: B - 1st stage of fractal analysis, S (box size) 1/2; C - 2nd stage of fractal analysis, S (box size) 1/4; D - 3rd stage of fractal analysis, S (box size) 1/8; E - 4th stage of fractal analysis, S (box size) 1/16; F - 5th stage of fractal analysis, S (box size) 1/32.

The minimum possible length of linear segments that can cover a linear contour on a digital bitmap image is 1 pixel. In the subsequent stages, as in the classic version of this method, the length of the linear segment is doubled several times. The modification developed by us allows to automate and simplify calculation by smoothing of a contour. Smoothing removes small bends from the contour that cannot be fractally covered with a radius larger than the radius of these bends. In the classical version of the Caliper method, those curves of the contour that have a radius less than the length of the fractal measure will not be covered by such linear segments. For example, if a contour has curves with a radius of 1 pixel, a fractal measure 2 pixels long will not cover those curves. Therefore, smoothing allows you to automatically modify the contour and get a result comparable to the classic version of the Caliper method. Gradual smoothing of the contour is performed starting from the second stage of fractal analysis using the tool "smoothing", followed by measuring the length of the contour after each smoothing. In the second stage, for images with a resolution of 128 pixels per inch, the anti-aliasing radius is 2 pixels, in the third stage - 4 pixels, in the fourth - 8 pixels, in the fifth - 16 pixels; the absolute length of the fractal segment (Sa) coincides with the smoothing radius and in the second stage of fractal analysis is 2 pixels, in the third - 4, in the fourth - 8 and in the fifth - 16 pixels (see Fig. 3) [15, 16].

When resizing an image, the smoothing radius and the absolute size of the fractal measure should be scaled in proportion to the change in resolution and image size. Fractional values that characterize the relative size of the fractal measure (S) do not depend on the image resolution and are in the first stage - 1/16, the second stage - 1/8, the third - 1/4, the fourth - 1/2, the fifth - 1. The number of fractal measures covering the studied contour (N) is defined as the ratio of P - contour length in pixels to S_a - the absolute size of the fractal measure in pixels: $N=P/S_a$ [15].

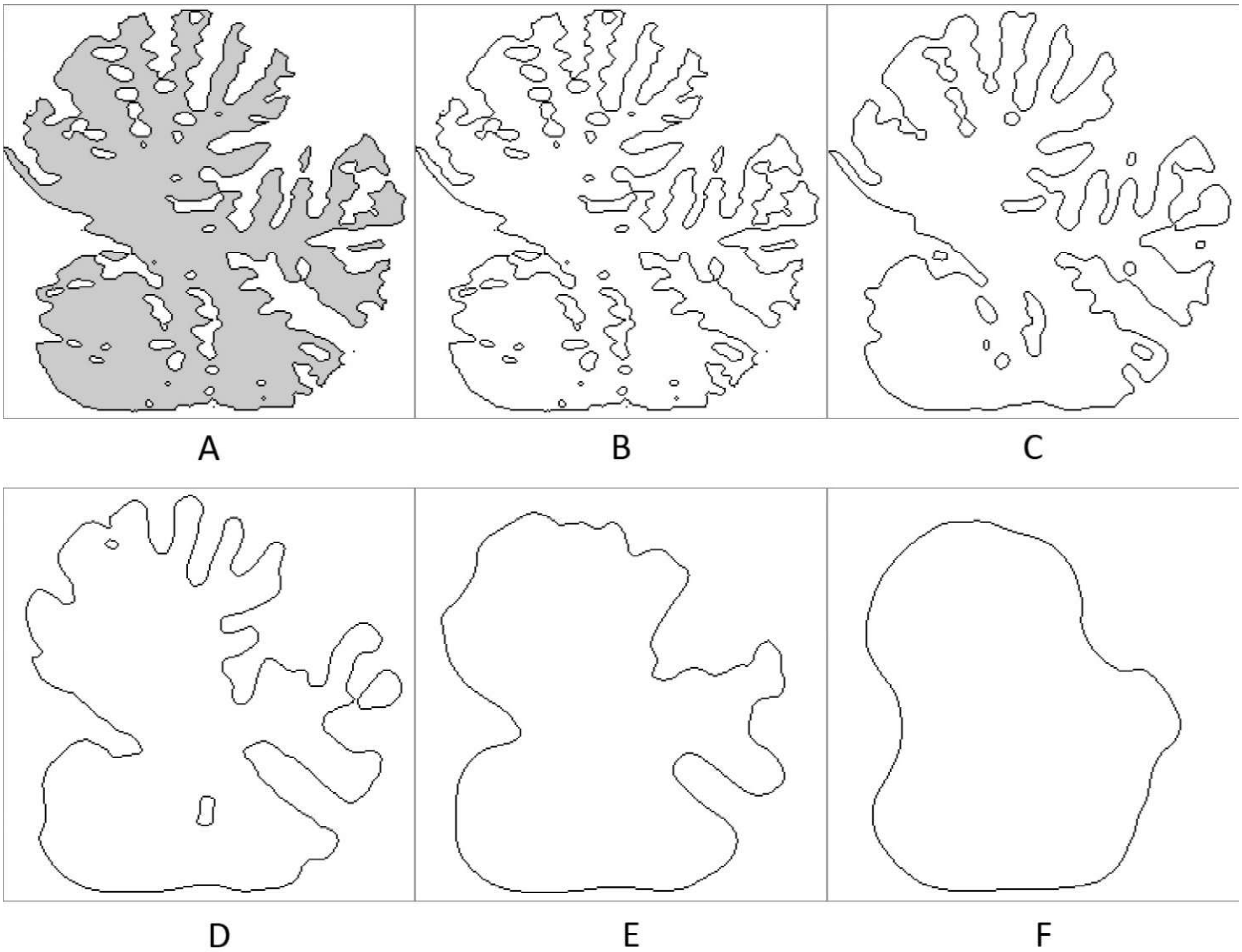


Fig. 3. Fractal analysis of the linear contour of the cerebellum using the Caliper method in the author's modification (described in [15]). A - selection of the linear contour of the object under study (contour delineation is used for clarity); B - 1st stage of fractal analysis, linear contour smoothing was not used, S=1; C-F - stages of fractal analysis using contour smoothing: C - 2nd stage of fractal analysis, S= 1/2; D - 3rd stage of fractal analysis, S=1/4; E - 4th stage of fractal analysis, S=1/8; F - 5th stage of fractal analysis, S=1/16. This modification was used by us for fractal analysis of the linear contour of the cerebellum [16].

Results

Initially, a fractal analysis of images with a resolution of 128 pixels per inch was performed. The values of the fractal dimension of the studied linear fractals, obtained using two methods of fractal analysis, did not match. Thus, the value of FD contour of the cerebellum, obtained using the method of Box counting, was 1.690, the method of Caliper - 1.501; the value of FD contour of the cortex of the cerebral hemispheres, obtained by the method of Box counting, was 1.125, the method of Caliper - 1.403. The values of the fractal dimension of these natural fractals are not known in advance, so in addition to these objects to validate the accuracy of measurements will be studied artificial fractal - Koch snowflake, the value of the fractal dimension of which is constant and therefore - known in advance (FD=Ln(4)/ Ln 3)≈1.26). The FD value of the Koch snowflake (fourth iteration) obtained using the Box counting method was 1.188, the Caliper method was 1.266. Therefore, the Caliper method in the study of images with a resolution of 128 pixels per inch allows to obtain a value of fractal dimension, which coincides with the true value of FD of the artificial linear fractal.

However, the resolution and size of the images used for analysis can vary significantly. Therefore, the influence of image size and resolution on fractal dimensional values was also studied. To do this, the same images were studied, but with three different resolution values: 128 pixels per inch (scale 1), 256 pixels per inch (scale 2) and 512 pixels per inch (scale 3) (Fig. 4). The dimensions of the images of the linear contour of the cerebellum and Koch's snowflake were 128x128 pixels (scale 1, small image size), 256x256 pixels (scale 2, medium size) and 512x512 pixels (scale 3, large size); the dimensions of the images with the linear contour of the cortex of the cerebral hemispheres were 512x400 pixels (scale 1, small image size), 1024x800 pixels (scale 2, medium size) and 2048x1600 pixels (scale

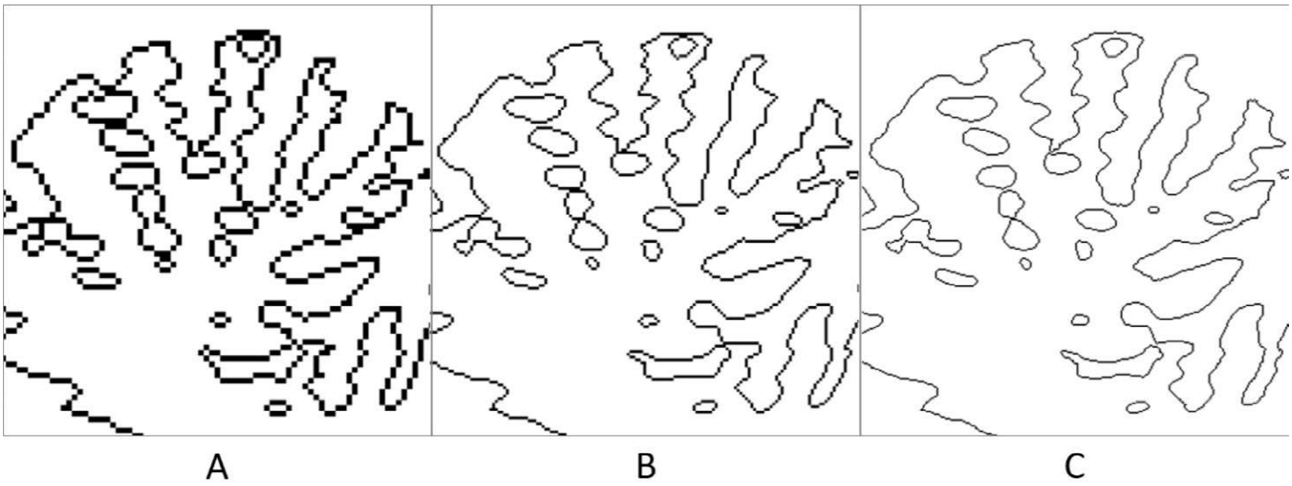
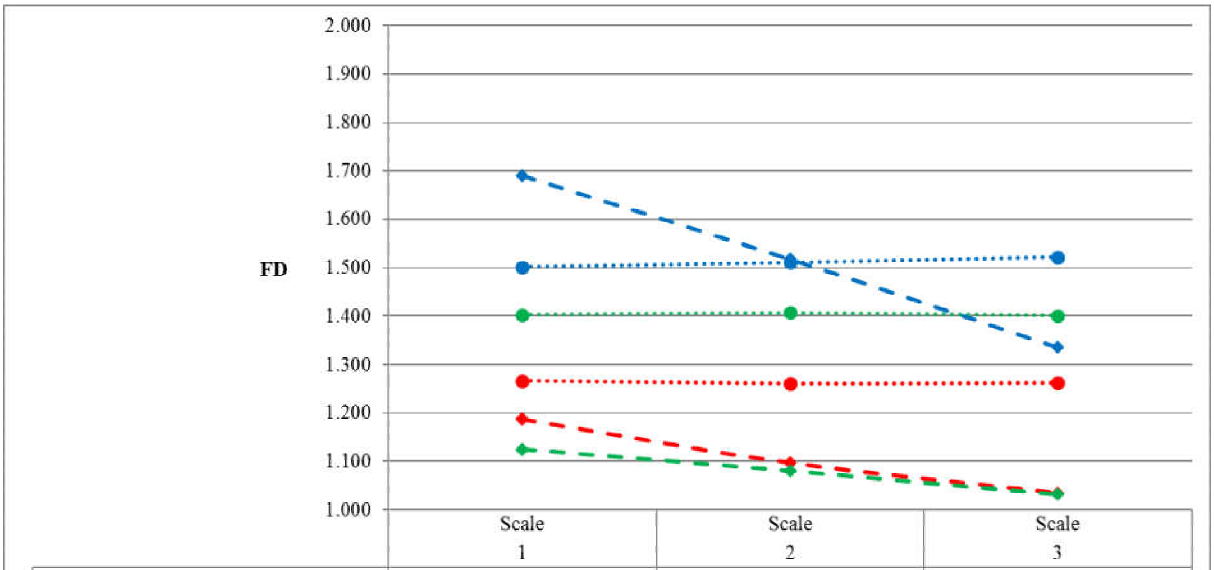


Fig. 4. Linear contour of the cerebellum in images with different resolutions: A - 128 pixels per inch, B - 256 pixels per inch, C - 512 pixels per inch. The outline of the line with the smallest possible value of width - 1 pixel is applied.



	Scale 1	Scale 2	Scale 3
Koch snowflake Box counting	1.188	1.097	1.035
Koch snowflake Caliper	1.266	1.261	1.262
Cerebellar vermis Box counting	1.690	1.518	1.336
Cerebellar vermis Caliper	1.501	1.511	1.522
Cerebral hemispheres Box counting	1.125	1.080	1.032
Cerebral hemispheres Caliper	1.403	1.407	1.401

Fig. 5. Fractal dimensional (FD) values determined on images with different resolutions and sizes using Caliper and Box counting methods. Scale 1 - resolution of 128 pixels per inch, scale 2 - resolution of 256 pixels per inch, and scale 3 - resolution of 512 pixels per inch.

3, large size). The smoothing radius values for the Caliper method and the absolute dimensions of the fractal measures for both fractal analysis methods changed in proportion to the change in the image resolution; the relative dimensions of fractal measures when scaling the image remained unchanged.

As can be seen from Figure 5, the FD values of the three studied fractals determined by the Box counting method decrease with increasing resolution and image size, and the FD values determined by the Caliper method remain virtually unchanged when these image parameters change. The FD values of the artificial fractal (Koch snowflakes) determined by the Caliper method on images of different sizes coincide with the true FD values of this fractal, in contrast to the FD values determined by the Box counting method. Therefore, the results obtained using the Caliper method are virtually independent of image size and resolution, which allows you to use this method to analyze linear contours on images of different sizes and with different resolutions.

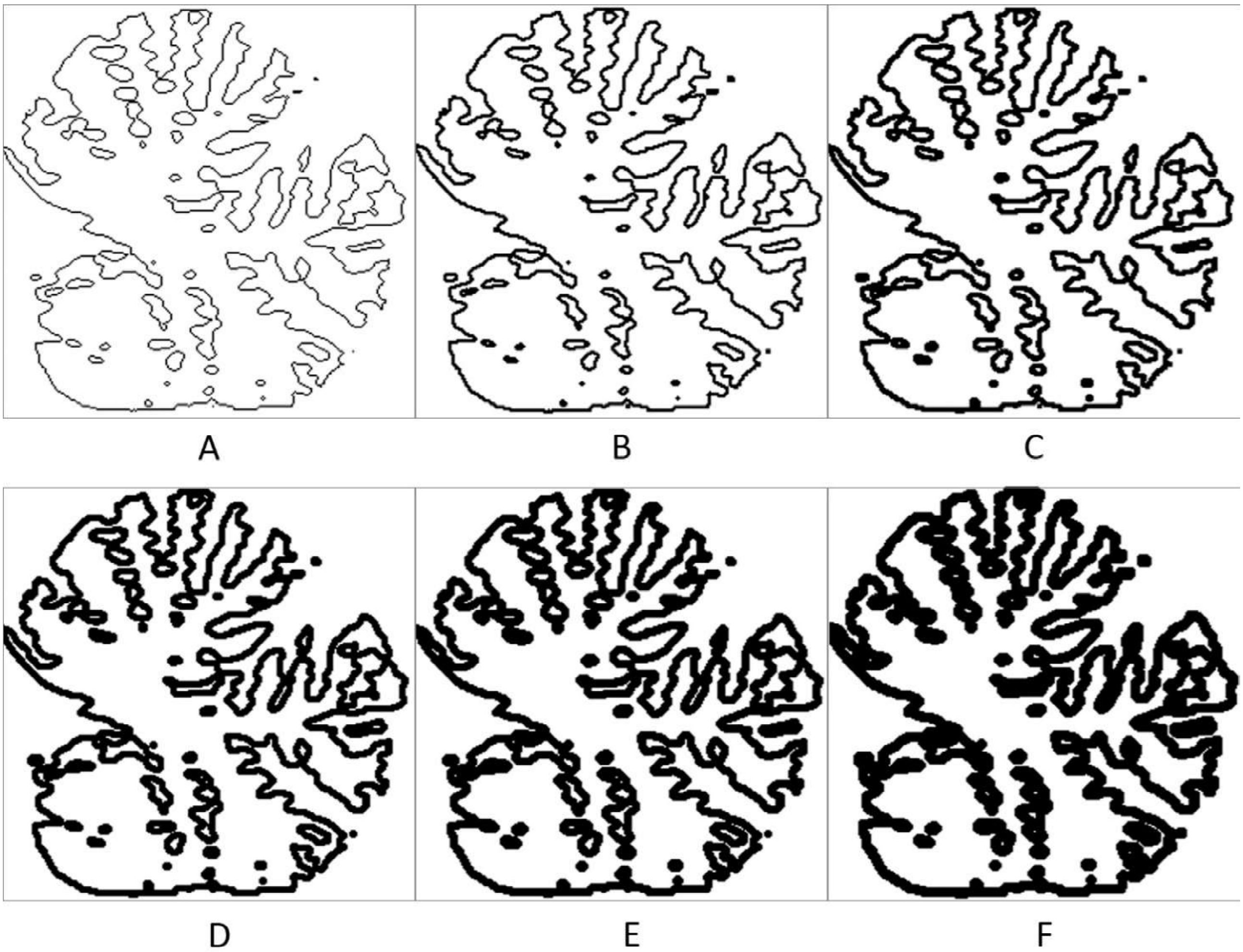


Fig. 6. Linear contour of the cerebellum, outline of the outer contour with a line of different width: A - 1 pixel; B - 2 pixels, C - 3 pixels, D - 4 pixels, E - 5 pixels, F - 6 pixels. Image resolution - 256 pixels per inch (image size 256x256 pixels).

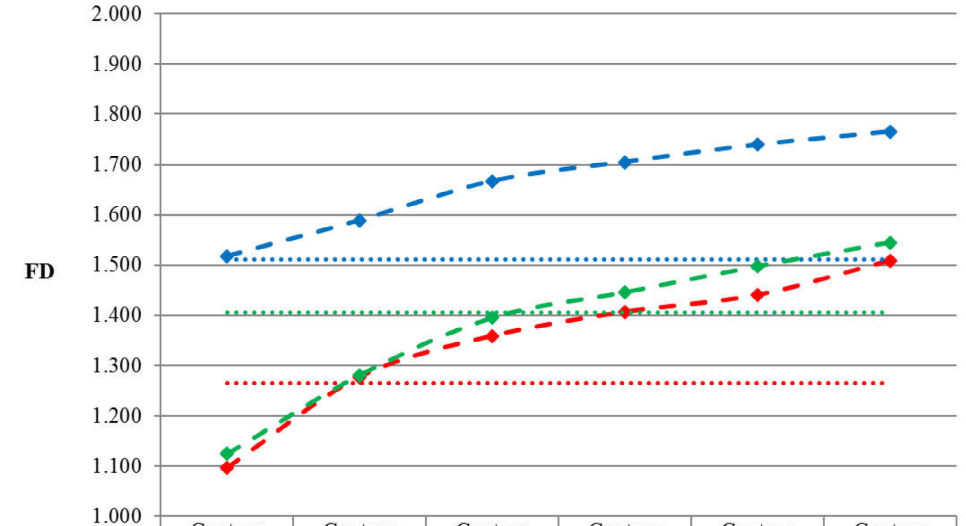
The difference in fractal dimension values determined by the Box counting method on images of different sizes and resolutions is due to the different relative widths of the contour line. For all three values of the resolution it is necessary to outline the contour with a line, using the minimum possible value - 1 pixel. However, for a 128x128 pixel image, 1 pixel will be 1/128 of the square of the image, for a 256x256 pixel image, 1 pixel will be 1/256 side, and for a 512x512 pixel image, 1/512 (see Figure 4). Therefore, under the same absolute width of the contour, the relative width of the contour decreases with increasing image size, which causes a difference in the values of the fractal dimension.

The next step was to study the effect of the absolute width of the line used to delineate the contour on the value of the fractal dimension. Because the Caliper method does not provide contour delineation, the effect of line width on fractal dimension values has only been studied for the Box counting method. During the preliminary preparation of the images, contours were drawn with lines from 1 to 6 pixels wide (Fig. 6). For the contour of the cerebellum and the Koch snowflake, the average image size was selected (256x256 pixels, resolution 256 pixels per inch, scale 2), for the contour of cerebral hemispheres - small size (512x400 pixels, resolution 128 pixels per inch, scale 1).

As can be seen from Figure 7, as the contour width increases, the fractal dimension of the three objects studied increases. That is, the value of the fractal dimension determined by the method of Box counting is affected not only by the complexity of the spatial configuration of the linear fractal, but also the width of the linear contour.

Discussion

Taking into account the results of comparative analysis of two methods of fractal analysis of linear contours of artificial and natural linear fractals, we can conclude that the FD values determined by Box counting are significantly influenced by both relative and absolute line widths of the studied linear fractal. Our conclusions are consistent with the data of King R.D., etc.: when performing fractal analysis



	Contour line width 1 pixel	Contour line width 2 pixels	Contour line width 3 pixels	Contour line width 4 pixels	Contour line width 5 pixels	Contour line width 6 pixels
Koch snowflake Box counting	1.097	1.278	1.360	1.408	1.441	1.509
Koch snowflake Caliper	1.265	1.265	1.265	1.265	1.265	1.265
Cerebellar vermis Box counting	1.518	1.590	1.668	1.706	1.740	1.765
Cerebellar vermis Caliper	1.511	1.511	1.511	1.511	1.511	1.511
Cerebral hemispheres Box counting	1.125	1.281	1.396	1.446	1.498	1.545
Cerebral hemispheres Caliper	1.407	1.407	1.407	1.407	1.407	1.407

Fig. 7. Fractal dimensional (FD) values determined on images with different linear contour widths using the Box counting method. For comparison, the FD values obtained using the Caliper method on images with identical size and resolution are given.

of the cerebral cortex using the method of Box counting, it was found that the thickness of the cortex has an effect on the value of FD, comparable to the effect of girification index, which characterizes the severity, complexity and number of convolutions of the cortex [8].

The method of Box counting in the two-dimensional version involves the use of a two-dimensional fractal measure - fractal squares. Therefore, the value of the fractal dimension is influenced not only by the length of the line, but also its width: the greater the width of the line, the more squares of the fractal grid this line will fall, the greater the value of the fractal dimension. Therefore, it is better to use the Box counting method to study objects that are close in configuration to the plane, or in cases where not only the length and complexity of the fractal line configuration but also its width must be taken into account. For example, atrophic changes in the cerebral cortex lead not only to smoothing the surface of the cortex, but also to reducing its thickness [7, 8].

Unlike the Box counting method, the Caliper method involves the use of a one-dimensional fractal measure that takes into account only one linear fractal size, namely its length. The width of the fractal line is not taken into account, which eliminates the effect of both the absolute width of the line and the relative width of the linear contour, which may differ in images of different sizes. Contour delineation is a necessary step in the preliminary preparation of images for the study of contours using the method of Box counting. But the images studied can have different quality, size and resolution, so the line width is difficult to standardize. The Caliper method does not require standardization of the image pre-algorithm and selection of the optimal line width, as contour delineation is not used. Therefore, the Caliper method is optimal for fractal analysis of linear fractals, especially in cases where you want to assess the complexity of the spatial configuration of the fractal line, leveling its width.

The two methods of fractal analysis analyzed in this work were used in our previous work to study the external linear contour of the cerebellum [16], fractal dimension values determined by two methods on 30 MR images of cerebellar vermis did not differ statistically significantly.

Both methods of fractal analysis in the classical version (Ruler (Caliper) and Box counting) were used to analyze the linear contours of benign and malignant breast tumors to interpret the results of mammography [17, 19], fractal dimension values obtained by two methods were close, but differed significantly in the contours of benign and malignant tumors, so both methods of fractal analysis allowed to reliably differentiate benign and malignant tumors.

In neuromorphological studies of the dendritic tree of neurons, a modified Richardson method (Caliper) was used in comparison with the Box counting method [20, 24].

In these studies, the dendritic tree of neurons was divided into linear segments and the analysis of each of the segments was performed by manual calculation, the fractal dimension of the dendritic tree as a whole was determined by the sum of measurements. The authors demonstrated that the Box counting method is sensitive to image orientation and to the presence or absence of skeletonization, and the Richardson method is independent of these factors and does not require the use of a grid for analysis [20]. The analysis of the dendritic tree of superficial and deep pyramidal neurons of the cerebral cortex of rats was performed; the fractal dimension of the dendritic tree of these neurons was statistically significantly different, but the level of statistical significance of the difference in FD values obtained by the modified Richardson method (Caliper) was significantly higher than the level of statistical significance of the difference in FD values obtained by Box counting [24].

Thus, taking into account the results of our research and the research of other scientists, we can conclude that the Caliper method is an effective method of mathematical image analysis in medicine, which has significant advantages in studying the linear contours of anatomical structures compared to the current Box counting method which is currently the most popular and used in the vast majority of studies.

Conclusion

1. Fractal analysis of linear fractals using the Caliper method allows to obtain values of fractal dimension that do not differ from the true value of the fractal dimension of artificial linear fractals; the values obtained using the Box counting method do not match the true values in some cases.

2. The values of the fractal dimension of linear fractals determined by the Caliper method do not depend on the size and resolution of the images. Fractal dimension values calculated using the Box counting method decrease as the image size and resolution increase.

3. The values of the fractal dimension of linear fractals, calculated using the method of Box counting, increase with increasing width of the linear contour; the values calculated using the Caliper method do not depend on the width of the contour line.

4. For fractal analysis of linear fractals (including linear contours of anatomical structures), preference should be given to the Caliper method and its modifications.

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ФРАКТАЛЬНИЙ АНАЛІЗ ЛІНІЙНИХ КОНТУРІВ АНАТОМІЧНИХ СТРУКТУР: АВТОРСЬКА МОДИФІКАЦІЯ СПОСОБУ CALIPER ПОРІВНЯНО ЗІ СПОСОБОМ BOX COUNTING

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Фрактальний аналіз дозволяє оцінити метричну розмірність та складність просторової конфігурації різних анатомічних структур, що дозволяє використовувати цей математичний метод для морфометрії у морфології та клінічній медицині. Для фрактального аналізу лінійних фрактальних об'єктів найчастіше використовують два способи фрактального аналізу: спосіб підрахунку квадратів (Box counting, Grid method) та спосіб Caliper (спосіб Річардсона, Perimeter stepping method, Ruler method, Divider dimension, Compass dimension, Yard stick method). Мета дослідження - порівняльний аналіз двох способів фрактального аналізу - способу Box counting та авторської модифікації способу Caliper для фрактального аналізу лінійних контурів анатомічних структур. Був проведений фрактальний аналіз трьох лінійних фракталів: штучного фракталу - сніжинки Коха та двох природніх фракталів - зовнішніх контурів піальної поверхні кори мозочка людини та кори великих півкуль головного мозку. Фрактальний аналіз проводився за допомогою способу Box counting та авторської модифікації способу Caliper. Значення фрактальної розмірності штучного лінійного фракталу (сніжинки Коха), отримані за допомогою способу Caliper, збігаються із істинним значенням фрактальної розмірності цього фракталу, але значення фрактальної розмірності, отримані за допомогою способу Box counting, із істинним значенням фрактальної розмірності не співпадають. Тому фрактальний аналіз лінійних фракталів за допомогою способу Caliper дозволяє отримати правдивіші результати, ніж спосіб Box counting. Значення фрактальної розмірності штучного та природних фракталів, обчислені за допомогою способу Box counting, зменшуються при збільшенні розміру та роздільної здатності зображення; при використанні способу Caliper значення фрактальної розмірності від цих параметрів зображення не залежать. Значення фрактальної розмірності лінійних фракталів, обчислені за допомогою способу Box counting, зростають при збільшенні ширини лінійного контуру; значення, обчислені за допомогою способу Caliper, не залежать від ширини лінії. Таким чином, для фрактального аналізу лінійних фракталів перевага має надаватися способу Caliper та його модифікаціям.

Ключові слова: *фрактальний аналіз, морфометрія, лінійний контур, Caliper, Box counting.*