

## Section: Chemistry

# VECTOR-BASED GRAPHICAL METHOD FOR BALANCING CHEMICAL REACTIONS

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The peculiarity of the vector approach for balancing chemical reactions is not only the simplicity and efficiency of mathematical calculations, but also the possibility of analysing and balancing chemical reactions without using mathematical methods.

This approach is based on compliance with the principles of conservation of chemical systems and the sum of vectors. In this context, each reagent is represented as a vector whose coordinates correspond to the number of atoms of each element.

The total number of atoms of each type in the reaction mixture corresponds to the sum of the values at each coordinate for all vectors, and the number of vectors of each type corresponds to the number of molecules in the reaction mixture

From this perspective, any chemical reaction can be interpreted graphically as two sets of vectors whose sums are equivalent. The vector approach enables rapid balancing of chemical reactions even in the absence of algebraic techniques, and leads to a range of important consequences and analytical opportunities.

### **A TWO-ELEMENT REACTION WITH A VALID SOLUTION**

Reaction scheme:  $C + O_2 \Rightarrow CO$

Reactants: C, O<sub>2</sub>

Products: CO

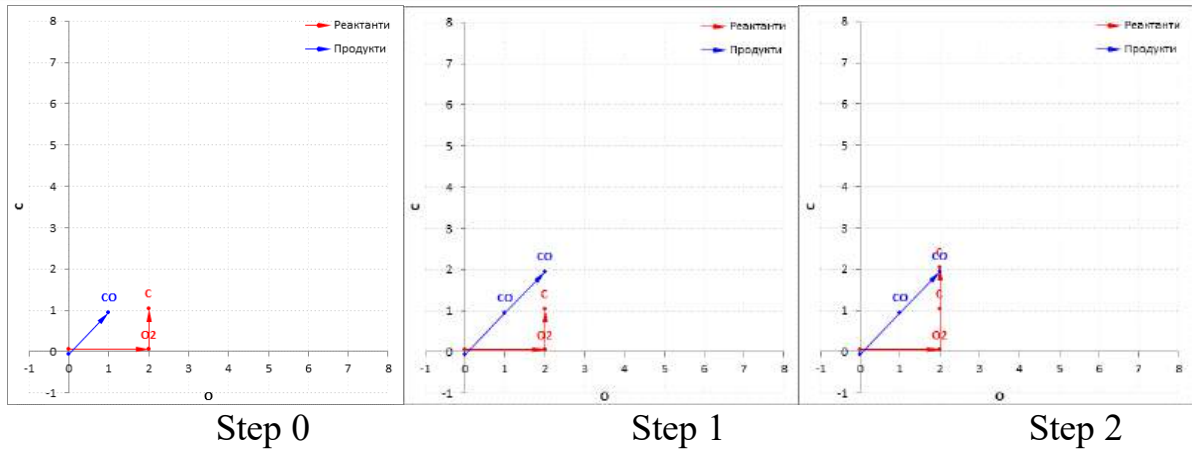
In vector form,  $R(n_C, n_O)$ :  $R_C = R(1, 0)$ ,  $R_{O_2} = R(0, 2)$ ,  $P_{CO} = P(1, 1)$

Solution sequence:

Step 0. A mixture of reactants (in red)  $R_C + R_{O_2}$  and products (in blue)  $P_{CO}$  is formed.

Step 1 – Insufficient O in the products, increase CO.

Step 2 – Insufficient C in the reactants, increase C.



Step 3 – Paths intersect — sum the vectors  $2R_C, 1R_{O_2}, 2R_{CO}$

Step 4 – Chemical equation:  $2C + O_2 = 2CO$

### A TWO-ELEMENT REACTION WITH A VALID SOLUTION

Reaction scheme:  $C + CO_2 \Rightarrow CO$

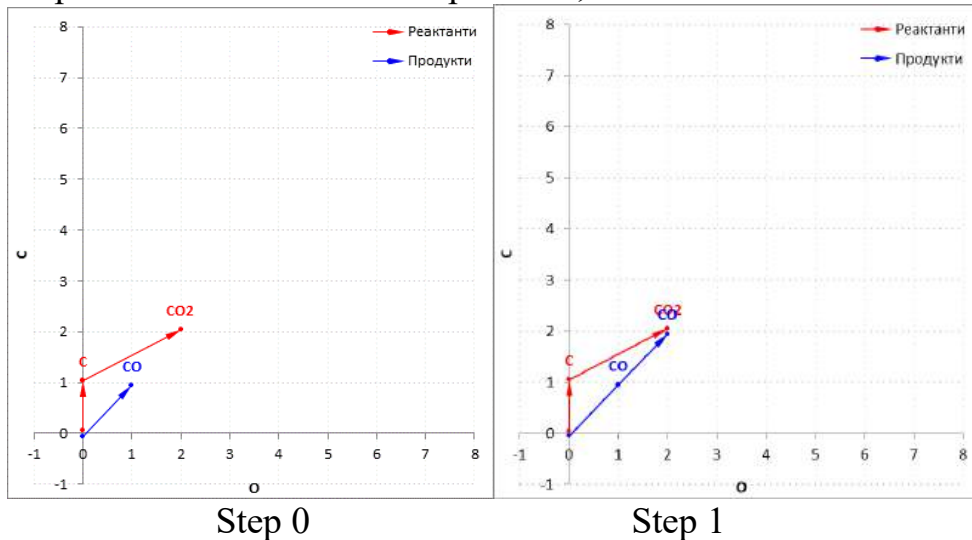
Reactants: C,  $CO_2$

Products: CO

In vector form,  $R(n_C, n_{O_2})$ :  $R_C = R(1,0)$ ,  $R_{CO_2} = R(1,2)$ ,  $P_{CO} = R(1,1)$

Step 0. A mixture of reactants (in red)  $R_C + R_{CO_2}$  and products (in blue)  $P_{CO}$  is formed.

Step 1 – Insufficient C in the products, increase  $CO_2$



Step 2 – Paths intersect — sum the vectors  $1R_C, 1R_{CO_2}, 2R_{CO}$

Step 3 – Chemical equation:  $C + CO_2 = 2CO$

### UNSOLVABLE TWO-ELEMENT REACTION

Reaction scheme:  $C + CO \Rightarrow CO_2$

Reactants: C, CO

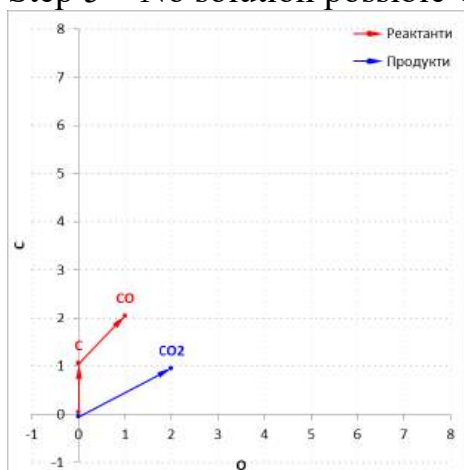
Products:  $CO_2$

In vector form,  $R(n_C, n_{CO})$ :  $R_C = R(1,0)$ ,  $R_{CO} = R(1,1)$ ,  $P_{CO_2} = R(1,2)$

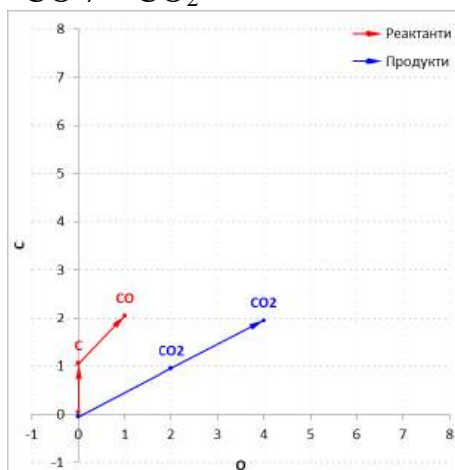
Step 0. A mixture of reactants (in red)  $R_C + R_{CO}$  and products (in blue)  $P_{CO_2}$  is formed.

Step 1 – Insufficient C in the products, increase CO<sub>2</sub>  
 Step 2 – Insufficient O in the reactants, increase CO  
 Step 3 – Insufficient C in the products, increase CO<sub>2</sub>  
 Step 4 – Conclusion: adding any vector increases the endpoint distance → stop search.

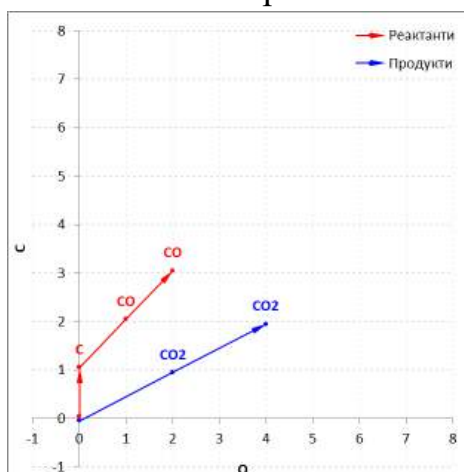
Step 5 – No solution possible  $C+CO \neq CO_2$



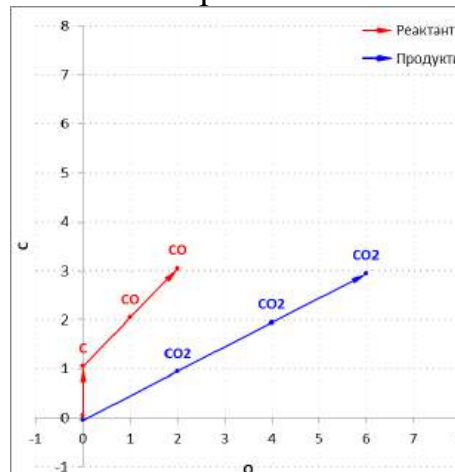
Step 0



Step 1



Step 2



Step 3

### TOTAL TWO-ELEMENT REACTION

Reaction scheme:  $O_2+C+CO \Rightarrow CO_2$

Reactants: O<sub>2</sub>, C, CO

Products: CO<sub>2</sub>

In vector form,  $R(n_C, n_O)$ :  $R_{O_2} = R(0,2)$ ,  $R_C = R(1,0)$ ,  $R_{CO} = R(1,1)$ ,  $P_{CO_2} = R(1,2)$

Step 0. A mixture of reactants (in red)  $R_C + R_{CO}$  and products (in blue)  $P_{CO_2}$  is formed.

Step 1 – Insufficient C in the products, increase CO<sub>2</sub>

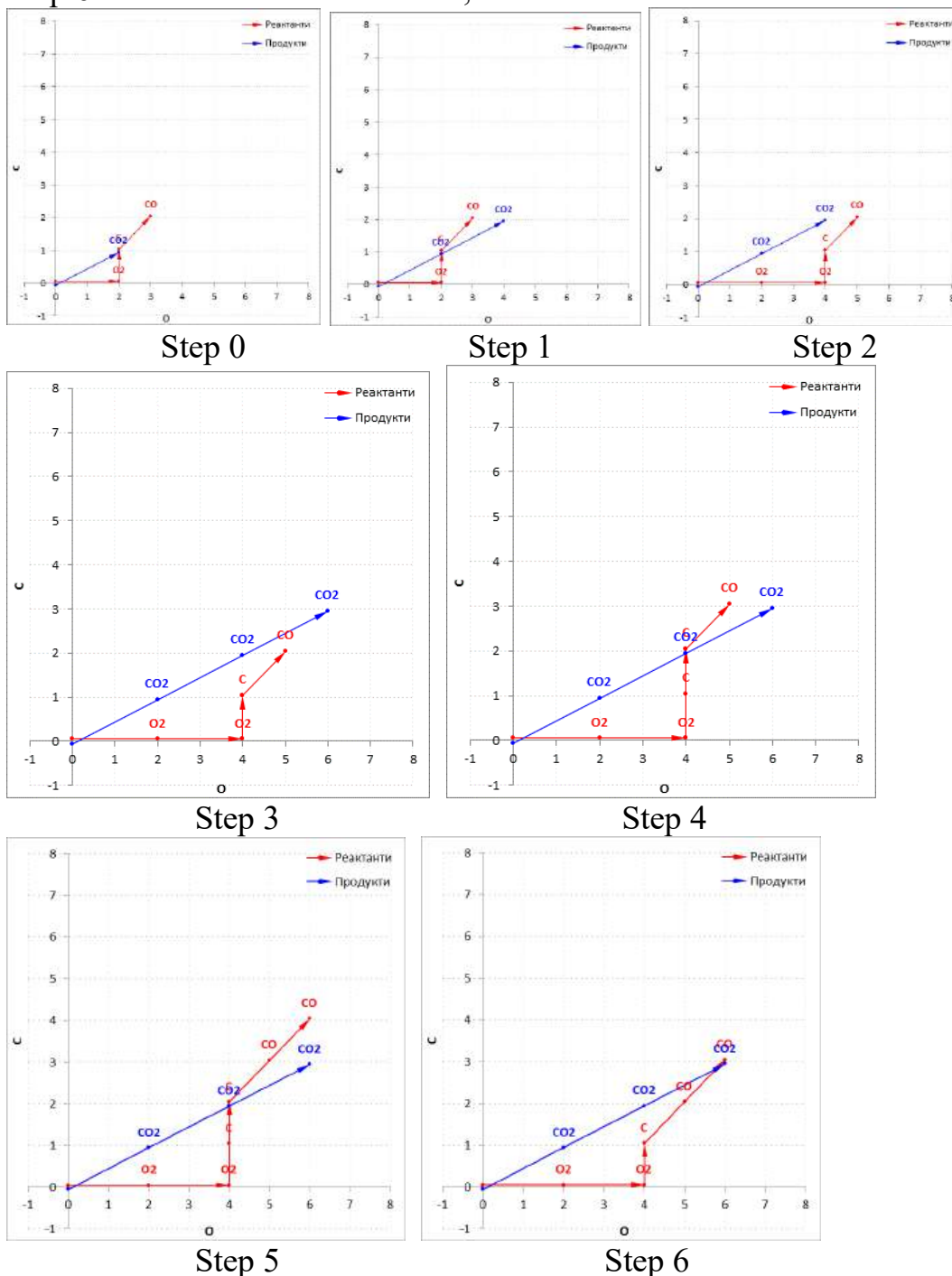
Step 2 – Insufficient O in the reactants, increase O<sub>2</sub>

Step 3 – Insufficient O in the products, increase CO<sub>2</sub>

Step 4 – Insufficient C in the reactants, increase C

Step 5 – Insufficient O in the reactants, increase CO

Step 6 – Excess C in the reactants, decrease C



Step 7 – Paths intersect — sum the vectors:  $2R_{O_2}$ ,  $R_C$ ,  $2R_{CO}$ ,  $3P_{CO_2}$

Step 8 – Chemical equation:  $2O_2 + C + 2CO = 3CO_2$

**•GENERAL METHOD FOR GRAPHICAL BALANCING (TWO-REACTANT CASE)**

- Step 0. Creation of Reactant Vectors
  - Elements involved in the reaction: C, O
  - Reagents present in the reaction mixture: C,  $O_2$ , CO,  $CO_2$
  - Reagents in vector form,  $R(n_C, n_O)$ :

- Reactants –  $R_C(1,0)$ ,  $R_{O_2}(0,2)$ ,  $R_{CO}(1,1)$ ,  $R_{O_2}(1,2)$
- Products –  $P_C(1,0)$ ,  $P_{O_2}(0,2)$ ,  $P_{CO}(1,1)$ ,  $P_{O_2}(1,2)$

○ Coordinate representation options:

- C (x-axis), O (y-axis)
- O (x-axis), C (y-axis)

Step 1. Identification of Reactant and Product Vectors

○ It is assumed that the mixture of reactants and products may contain any of the Reagents, but:

- A reagent cannot simultaneously be a reactant and a product
- The overall path may consist of smaller paths
- For simplification, assume that the products are more complex (contain more atoms and thus have a longer vector)

○ The reactants and products must contain the same set of chemical elements

Step 2. Formation of Possible Vector Combinations (the number of vectors on one side must be at least one less than the total number of available vectors, and the number of vectors in the combination can be more than 1)

▪ For one reagent

- C
- $O_2$
- CO
- $CO_2$

▪ For two reagents

- C,  $O_2$
- C, CO
- C,  $CO_2$
- $O_2$ , CO
- $O_2$ ,  $CO_2$
- CO,  $CO_2$

▪ For three reagents

- C,  $O_2$ , CO
- C,  $O_2$ ,  $CO_2$
- C, CO,  $CO_2$
- $O_2$ , CO,  $CO_2$

Step 3. To formulate possible transformation schemes, we consider each of the combinations as reactants and select another combination that does not contain the reactants from the first combination and analyse them for stoichiometric constraints.

○ For one reactant:

- $C \not\Rightarrow O_2, CO, CO_2$  – elements do not match
- $O_2 \not\Rightarrow C, CO, CO_2$  – elements do not match
- $CO \Rightarrow C, O_2, CO_2$

- $\text{CO} \Rightarrow \text{C}, \text{O}_2$
  - $\text{CO} \Rightarrow \text{C}, \text{CO}_2$
  - $\text{CO} \not\Rightarrow \text{O}_2, \text{CO}_2$  – The O/C ratio is not satisfied
  - $\text{CO} \not\Rightarrow \text{CO}_2$  – The O/C ratio is not satisfied
  - $\text{CO}_2 \Rightarrow \text{C}, \text{O}_2, \text{CO}$
  - $\text{CO}_2 \Rightarrow \text{C}, \text{O}_2$
  - $\text{CO}_2 \Rightarrow \text{C}, \text{CO}$
  - $\text{CO}_2 \Rightarrow \text{O}_2, \text{CO}$
  - $\text{CO}_2 \not\Rightarrow \text{CO}$  – The O/C ratio is not satisfied
  - For two reactants:
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}, \text{CO}_2$
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}$
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}_2$
    - $\text{C}, \text{CO} \not\Rightarrow \text{O}_2, \text{CO}_2$  – The O/C ratio is not satisfied
    - $\text{C}, \text{CO} \Rightarrow \text{CO}_2$
    - $\text{C}, \text{CO} \not\Rightarrow \text{O}_2$  – C missing on the right
    - $\text{C}, \text{CO}_2 \not\Rightarrow \text{O}_2, \text{CO}$  – The O/C ratio is not satisfied
    - $\text{C}, \text{CO}_2 \Rightarrow \text{CO}$
    - $\text{C}, \text{CO}_2 \not\Rightarrow \text{O}_2$  – C missing on the right
    - $\text{O}_2, \text{CO} \not\Rightarrow \text{C}, \text{CO}_2$  – The O/C ratio is not satisfied
    - $\text{O}_2, \text{CO} \not\Rightarrow \text{C}$  – O missing on the right
    - $\text{O}_2, \text{CO} = \text{CO}_2$
    - $\text{O}_2, \text{CO}_2 \not\Rightarrow \text{C}, \text{CO}$  – The O/C ratio is not satisfied
    - $\text{CO}, \text{CO}_2 = \text{C}, \text{O}_2$
  - For three reactants:
    - $\text{C}, \text{O}_2, \text{CO} \Rightarrow \text{CO}_2$
    - $\text{C}, \text{O}_2, \text{CO}_2 \Rightarrow \text{CO}$
    - $\text{C}, \text{CO}, \text{CO}_2 \not\Rightarrow \text{O}_2$  – C missing on the right
    - $\text{O}_2, \text{CO}, \text{CO}_2 \not\Rightarrow \text{C}$  – O missing on the right
- Step 4. Retain only theoretically feasible reactant combinations
- For one reactant:
    - $\text{CO} \Rightarrow \text{C}, \text{O}_2, \text{CO}_2$
    - $\text{CO} \Rightarrow \text{C}, \text{O}_2$
    - $\text{CO} \Rightarrow \text{C}, \text{CO}_2$
    - $\text{CO}_2 \Rightarrow \text{C}, \text{O}_2, \text{CO}$
    - $\text{CO}_2 \Rightarrow \text{C}, \text{O}_2$
    - $\text{CO}_2 \Rightarrow \text{C}, \text{CO}$
    - $\text{CO}_2 \Rightarrow \text{O}_2, \text{CO}$
  - For two reactants:
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}, \text{CO}_2$
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}$
    - $\text{C}, \text{O}_2 \Rightarrow \text{CO}_2$

- $C, CO \Rightarrow CO_2$
- $C, CO_2 \Rightarrow CO$
- $O_2, CO = CO_2$
- $CO, CO_2 = C, O_2$
- For three reactants:
  - $C, O_2, CO \Rightarrow CO_2$
  - $C, O_2, CO_2 \Rightarrow CO$

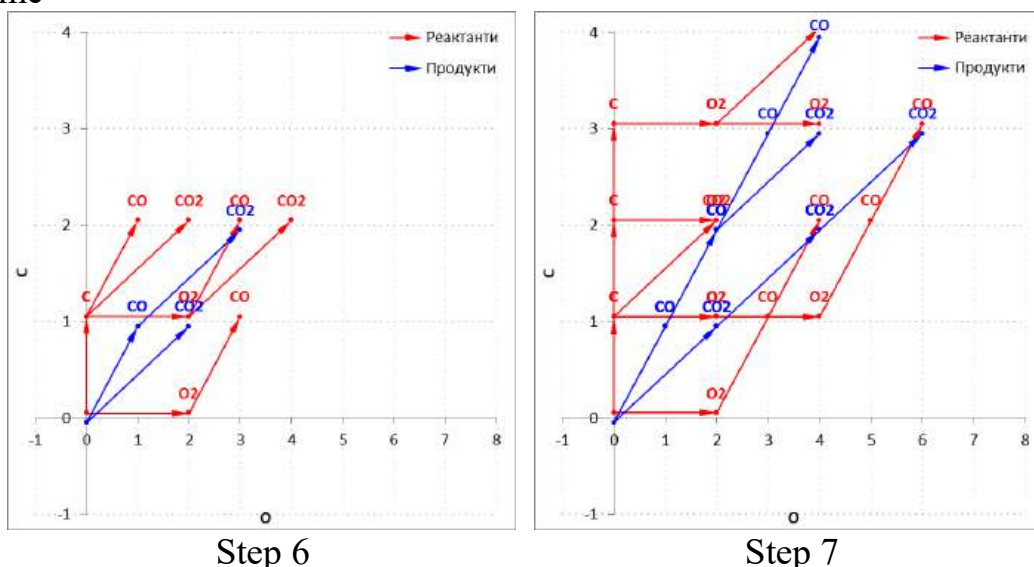
Step 5. Determine direct and inverse combinations - a direct combination is a combination where the number of reactants is greater and the size of the product vector is larger (formation reaction):

- $CO \Rightarrow C, O_2, CO_2$
- $C, O_2, CO_2 \Rightarrow CO$
  
- $CO \Rightarrow C, O_2$
- $C, O_2 \Rightarrow CO$
  
- $CO \Rightarrow C, CO_2$
- $C, CO_2 \Rightarrow CO$
  
- $CO_2 \Rightarrow C, O_2, CO$
- $C, O_2, CO \Rightarrow CO_2$
  
- $CO_2 \Rightarrow C, O_2$
- $C, O_2 \Rightarrow CO_2$
  
- $CO_2 \Rightarrow O_2, CO$
- $O_2, CO = CO_2$
  
- $C, O_2 \Rightarrow CO, CO_2$
- $CO, CO_2 = C, O_2$

Step 6. Only direct reactions are retained

- $C, O_2, CO_2 \Rightarrow CO$
- $C, O_2 \Rightarrow CO$
- $C, CO_2 \Rightarrow CO$
- $C, O_2, CO \Rightarrow CO_2$
- $C, O_2 \Rightarrow CO_2$
- $O_2, CO \Rightarrow CO_2$
- $C, O_2 \Rightarrow CO, CO_2$

Step 7. Change the number of vectors until the paths coincide for each reaction scheme



- Step 8. Calculate the number of vectors of each type
- Step 9. Write the solution in the form of a traditional chemical equation
  - $3C + O_2 + CO_2 = 4CO$
  - $2C + O_2 = 2CO$
  - $C + CO_2 = 2CO$
  - $C + 2O_2 + 2CO = 3CO_2$
  - $C + O_2 = CO_2$
  - $O_2 + 2CO = 2CO_2$
  - $3C + 2O_2 = CO + CO_2$

Thus, as illustrated by the examples provided, graphical balancing is relatively simple and can be extended to an arbitrary number of reactants and elements. Moreover, it clearly demonstrates the multiplicity of possible pathways leading to the same product — a feature that traditional mathematical methods do not typically reveal.

### References

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