V. N. Karazin Kharkiv National University
Szczecin University

DECT-2018

3rd International Scientific Conference

DIFFERENTIAL EQUATIONS and CONTROL THEORY

September 25–27, 2018

Book of Abstracts

Kharkiv 2018

3rd International Scientific Conference "DIFFERENTIAL EQUATIONS and CONTROL THEORY (DECT-2018)": Book of abstracts, Kharkiv: V.N. Karazin Kharkiv National University, 2018, 52 p.

Затверджено на засіданні кафедри прикладної математики
(протокол № 8 від 27.08.2018)

Друкується за рішенням Вченої Ради факультету математики і інформатики Харківського національного університету імені В. Н. Каразіна
(протокол № 7 від 27.08.2018)

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3rd International Conference "Differential Equations and Control Theory" is co-financed under the agreement No. 776/P-DUN/2018 from funds of the Minister of Science and Higher Education of the Republic of Poland allocated to science dissemination activities

of exponentials. This allows to construct steering controls and solve moment problem for each state of the model space.

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Nano thermo-hydrodynamics models for quantitative estimations of the cell membrane fluidity: a review

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The cells are the smallest units of the live matter, and their interaction with other cells and environment are determined by the cellular membranes. The latter possess mechanical, thermal and electric properties, which values strongly depend on the state of the cells (healthy, influenced, stresses, diseased). That is why the mathematical models of the cellular membrane and their physical properties are essential for the medical diagnostics purposes [1]. The mechanical properties of the cells and their membranes are represented by their density, elasticity and fluidity. While the density can be easily measured; the elasticity can be estimated by the rheometry and micro/nano indentometry; but the measurements of fluidity needs more complex mathematical models and governing equations for the heat transfer [2]. The most relevant model of the heat transfer at the micro and nano scales is based on the Navier-Stokes equations for the incompressive fluid (water as the main components of the cells and their membranes) combined by the heat transfer equation in the Guyer-Krumhansl form [3]

$$\tau\frac{\partial^2 T}{\partial t^2}+\frac{\partial T}{\partial t}+\rho_b c_b w_b(T-T)_b)=k\left(\nabla^2 T+a\frac{\partial}{\partial t}\nabla^2 T\right)+q_m+q_e, \qquad (1)$$

where T is the temperature, τ is the relaxation time, a is the diffusivity, k is the thermal conductivity, q_m and q_e are the methabolic and externally stimulated sources of heat, the subscript b related to the blood flow in the tissues provided the cells in the perfused tissue or bioreactor are considered.

The system of the Navier-Stokes equations together with the heat equation in the form (1) has been solved by the finite difference method with iterations over time.

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On synthesis problem for inherently nonlinear systems

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We study the controllability problem for a class of nonlinear systems of the form

$$\begin{cases} \dot{x}_1 = u, \quad |u(x)| \le d, \\ \dot{x}_i = x_{i-1}^{2k_{i-1}+1} + f_{i-1}(t,x,u), \quad i = 2, \dots, n, \end{cases} \tag{1}$$

where $u \in \mathbb{R}$ is a control, $d > 0$ is a given number, $k_i = \frac{p_i}{q_i}$ ($p_i > 0$ is an integer, $q_i > 0$ is an odd integer), $f_i(t,x,u)$ $(i = 1, \dots, n-1)$ are continuous real-valued functions with $f_i(t,0,0) = 0$ for all $t \ge 0$.

We construct a class of bounded controls $u = u(x)$ such that for any initial point $x_0 \in U(0)$ the solution $x(t,x_0)$ of the corresponding closed-loop system is well-defined on the interval $[0, T(x_0)]$ and ends at 0 in a finite-time $T(x_0) < +\infty$, i.e. $\lim_{t \to T(x_0)} x(t,x_0) = 0$.

The class of smooth stabilizing controls for system (1) was proposed in [1]. The synthesis problem for the case when $f_i(t,x,u) = 0$ $(i = 1, \dots, n-1)$ and $k_i = 0$ $(i = 1, \dots, n-2)$, $k_{n-1} > 0$ was solved in [2]. The approach which was proposed in [2] for constructing finite-time stabilizers is based on the controllability function method [3]. Under some additional growth conditions imposed on functions $f_i(t,x,u)$ we develop this approach to construct a class of bounded finite-time stabilizing controls $u = u(x)$ for system (1). To this end, we construct a class of controllability functions $\Theta(x)$ such that the inequality $\dot{\Theta}(x) \le -\beta \Theta^{1-\frac{1}{\alpha}}(x)$ holds for some $\alpha \ge 1$, $\beta > 0$. The former inequality guarantees that any trajectory of the closed-loop system starting in $U(0)$ hits the origin in some finite time $T(x_0)$.

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